



Nonparametric estimation of market risk: an application to agricultural commodity futures

Agricultural
commodity
futures

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Abstract

Purpose – While the extant literature is replete with theoretical and empirical studies of value at risk (VaR) methods, only a few papers have applied the concept of VaR to quantify market risk in the context of agricultural finance. Furthermore, papers that have done so have largely relied on parametric methods to recover estimates of the VaR. The purpose of this paper is to assess extreme market risk on investment in three actively traded agricultural commodity futures.

Design/methodology/approach – A nonparametric Kernel method was implemented which accommodates fat tails and asymmetry of the portfolio return density as well as serial correlation of the data, to estimate market risk for investments in three actively traded agricultural futures contracts: corn, soybeans, and wheat. As a futures contract is a zero-sum game, the VaR for both short and long sides of the market was computed.

Findings – It was found that wheat futures are riskier than either corn or soybeans futures over both periods considered in the study (2000-2008 and 2006-2008) and that all three commodities have experienced a sharp increase in market risk over the 2006-2008 period, with VaR estimates 10-43 percent higher than the long-run estimates.

Research limitations/implications – Research is based on cross-sectional data and does not allow for dynamic assessment of expenditure elasticities

Originality/value – This paper differs methodologically from previous applications of VaR in agricultural finance in that a nonparametric Kernel estimator was implemented which is exempt of misspecification risk, in the context of risk management of investment in agricultural futures contracts. The application is particularly relevant to grain elevator businesses which purchase grain from farmers on a forward contract basis and then turn to the futures markets to insure against falling prices.

Keywords United States of America, Agriculture, Futures markets, Value analysis, Contracts

Paper type Research paper

1. Introduction

The increase in the number and trading volume of derivative instruments in recent years has spurred the development of sophisticated measures of market risk. One of the most popular market risk metrics among financial managers is value at risk (VaR). The popularity of VaR as a risk measure stems from two main reasons. First, VaR is relatively easy to understand and interpret, it is simply the maximum dollar amount or rate of return a portfolio can lose over a specified time interval with a certain degree of confidence. For example, a one week 99 percent VaR of 5 percent means that there is a 99 percent probability that the portfolio will not experience a loss greater than 5 percent of its current value over the next trading week. Losses greater than 5 percent are suffered only under “unusual” market conditions which in this example occur with a probability of 1 percent. Second, the development of an open-source VaR methodology in 1994 by Riskmetrics, a division of JP Morgan, fueled significant growth in the use of VaR as a

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market risk metric. The increasing reliance of financial institutions on VaR and its simplicity led the Basel Committee on Banking Supervision as well as the Federal Reserve in 1995 to allow banks to use their internal VaR to determine capital requirements for market risk (Linsmeier and Pearson, 1996).

VaR is often estimated by the parametric variance-covariance method. The variance-covariance (delta-normal) method assumes that the underlying distribution of portfolio values is normal. However, empirical evidence indicates that financial returns exhibit thick tails which are inconsistent with the normal distribution. This has led to the use of the *t*-distribution which has fatter tails than the normal (e.g. Lin and Shen, 2006) and the extreme value theory which models only the tail of the portfolio distribution using the generalized Pareto distribution (Odening and Hinrichs, 2003; Lin and Shen, 2006)[1].

Many studies evaluate the VaR for the purpose of financial risk management; however, only a few studies have done so to measure market risk for agriculture-related investments. Boehlje and Lins (1998) identify the VaR as a potential measure of market risk for agricultural industries in light of a changing risk environment. Manfredo and Leuthold (2001) use the variance-covariance and historical simulation approaches to investigate the market risk in cattle feeding; Odening and Hinrichs (2003) use the extreme value theory to estimate the VaR for the German hog market. In a case study, Wilson *et al.* (2007) illustrate how agricultural commodity processors can use VaR to manage price risk.

All of these applications of VaR to agricultural risk management share the feature of parametric modeling. The parametric approach – such as the delta normal – has the advantages of ease of implementation and closed-form formula for the VaR estimator; the obvious downside is the potential for model misspecification which may result in poor appreciation of risk.

This paper differs methodologically from the previous applications of VaR to agricultural finance in that we implement a nonparametric Kernel VaR model (Chen and Tang, 2005) that is immune to distributional misspecification to assess extreme market risk on investment in three actively traded agricultural commodity futures. The use of VaR to manage market risk of agricultural futures contracts is particularly relevant for grain elevator operators such as Archer Daniels Midland (ADM), Cargill, and local cooperatives who buy grain from farmers and then seek to insure against the risk of falling grain prices by selling futures contracts. Knowledge of the VaR for their investment in futures contracts can help these operators decide whether to increase or reduce the short hedge position. The VaR can also be instrumental in predicting margin calls since it is an estimate of the maximum loss on an investment over the next trading period.

The nonparametric Kernel method uses the weighted average of the order statistics around the quantile of interest as the VaR estimate. The resulting estimate is distribution-free and consistent for independent or dependent return series. Because the Kernel method consists of local weighted averaging of the data, it requires a large dataset to achieve consistency. Unfortunately data are usually sparse near the boundaries which may result in imprecise VaR estimates. To circumvent the data limitation problem and its implication for the accuracy of the nonparametric VaR estimate, we propose to exploit the panel nature of futures data. Specifically, for each commodity, we pool the returns of several futures contracts instead of relying on one single time series of returns based on the shortest contract as commonly done. This has the effect of increasing the number of observations in the tails which are the relevant data for the purpose of VaR computation.

On average in our sample, the four contracts with the largest open interest account for more than 92 percent of all underwritten contracts.

The remainder of the paper is organized as follows. Section 2 presents the nonparametric Kernel VaR estimator which is the focus of the paper. In section 3, the nonparametric Kernel VaR method is used to quantify market risk for investments in three key commodity futures contracts. Section 4 concludes.

2. Nonparametric estimation of VaR

Consider a continuously marked-to-market portfolio over the period $[1, T]$ with prices $\{P_t\}_{t=1}^T$ and let $r_t = \log(P_t/P_{t-1}) \times 100$ be the continuously compounded rate of return. Suppose $\{r_t\}_{t=1}^T$ is a strictly stationary process with a cumulative distribution function F . Then the $(1 - p)$ percent VaR, denoted VaR_p , is defined as:

$$VaR_p = \inf\{r : F(r) \geq p\}$$

where r is a support point and p a small positive number. Therefore the calculation of VaR consists of finding a quantile of the portfolio distribution based on the desired $(1 - p)$ percent level of confidence. The variance-covariance procedure assumes the underlying cumulative distribution F is normal. Statistical properties of the normal distribution are then used to determine the loss that will be equaled or exceeded p percent of the time, i.e. the VaR. The formula for the variance-covariance VaR is:

$$\widehat{VaR}_p = \Phi^{-1}(p)\sigma \quad (1)$$

where σ is the standard deviation of portfolio returns and $\Phi(\cdot)$ the cumulative distribution of the standard normal. For instance, the 95 percent VaR is equal to 1.96 times the standard deviation of portfolio returns. This closed-form solution for the VaR represents the biggest advantage of the variance-covariance approach over competing alternatives. For a desired level of confidence $(1 - p)$ percent, the only element of the VaR that is unknown is the portfolio volatility σ . Several methods have been proposed in the finance literature to estimate the components of the portfolio volatility. These methods include long-run historical average, historical moving average, generalized autoregressive conditional heteroscedasticity (GARCH), exponentially weighted average, and implied volatility.

The normality assumption about the distribution of returns used to construct the variance-covariance VaR estimator has been the subject of much debate among academics and practitioners because of the strong evidence that financial return distributions have thicker tails than the normal and may exhibit asymmetry (see e.g. Brooks *et al.*, 2005). To account for the excess kurtosis, academics have suggested the use of alternative cumulative distributions such as the t distribution or the generalized Pareto distribution. None however relieves the main source of concern which is misspecification risk and its solvency implications stemming from a poor assessment of capital reserves. Drawing from recent developments in the theoretical literature of VaR modeling (Gourieroux *et al.*, 2000; Chen and Tang 2005), this paper implements a distribution-free (nonparametric Kernel) method which yields a consistent estimate of the VaR even when the data are serially dependent.

Let $K(u)$ be a smooth, positive, real-valued function satisfying the conditions $\int K(u)du = 1$, $\int uK(u)du = 0$, and $\int u^2K(u)du < \infty$. The nonparametric kernel estimator of the cumulative distribution functions of the portfolio returns $F(r)$ is given

by:

$$\widehat{F}(r) = \frac{1}{T} \sum_{t=1}^T G\left(\frac{r - r_t}{h}\right)$$

where $G(u) = \int^u K(r)dr$ and h is a bandwidth that controls the smoothness of the Kernel estimate. Therefore the kernel estimator of VaR (Gourieroux *et al.*, 2000) denoted \widehat{VaR}_p satisfies:

$$\frac{1}{T} \sum_{t=1}^T G\left(\frac{\widehat{VaR}_p - r_t}{h}\right) = p \quad (2)$$

Chen and Tang (2005) investigate the asymptotic and finite sample properties of the Kernel VaR estimator. Under their assumptions 1-4 and as the sample size $T \rightarrow \infty$, the bias and variance of the kernel VaR estimator are[2]:

$$E \widehat{VaR}_p = VaR_p - \frac{1}{2} h^2 \sigma_K^2 f'(VaR_p) f^{-1}(VaR_p) + o(h^2)$$

$$Var(\widehat{VaR}_p) = \frac{1}{T} f^{-2}(VaR_p) \sigma_h^2(p; T) - \frac{2}{T} h f^{-1}(VaR_p) b_K + o\left(\frac{h}{T}\right)$$

where $f(r)$ is the underlying portfolio density function, $b_K = \int uK(u)G(u)du$, $\sigma_h^2(p; T) = \left\{ p(1-p) + 2 \sum_{k=1}^{T-1} (1-k/T) \gamma_h(k) \right\}$ and $\gamma_h(k) = cov\{G(VaR_p - r_1/h), G(VaR_p - r_{k+1}/h)\}$ Since the smoothing parameter h goes to zero as the sample size T goes to infinity, it follows from these statistical results that the Kernel VaR estimator is biased but consistent for strictly stationary and α -mixing return series $\{r_t\}_{t=1}^T$. Chen and Tang (2005) further derive the asymptotic normality of \widehat{VaR}_p :

$$\sqrt{T}(\widehat{VaR}_p - VaR_p) \xrightarrow{d} N(0, \sigma^2(p) f^{-2}(VaR_p))$$

where $\sigma^2(p) = \lim_{T \rightarrow \infty} \sigma_h^2(p; T)$. This result can be used to construct asymptotic confidence intervals for the Kernel VaR estimator.

3. Empirical study

Futures contracts are bought and sold for the purpose of price risk management by farmers and firms who wish to hedge the price risk of agricultural inputs or outputs. Agricultural commodities are characterized by considerable price fluctuations that emanate from several factors such as unfavorable weather conditions, natural disasters (e.g. hurricanes), shifts in local and global demand (due for example to changes in agricultural policy). The use of a commodity futures contract, a contract to buy or sell a commodity at a specified date in the future and at an agreed upon price can help producers, grain elevator operators, commodity processors, and other agribusinesses cushion the adverse effects of these price fluctuations. For example, grain elevator

operators such as ADM and Bunge purchase grain from farmers on a forward contract basis and then attempt to hedge the risk of falling prices by selling futures contracts for the same quantity of grain[3]. Because futures contracts are margined daily, hedgers stand to suffer significant losses when margin calls are triggered by unexpected sharp price movements as was the case in 2008[4]. Therefore a grain elevator operator may use the trend of daily VaRs of his portfolio to decide whether to offset his contracts or sell additional contracts. Longer-horizon VaRs, say weekly or monthly, can be used by an operator to assess potential portfolio losses over the next trading week or month and therefore potential capital requirements to meet marginal calls. This gives the short hedger time to negotiate for an increased line of credit if the assessment is that prices are going to rise further[5]. Additionally, VaR can be used to reduce hedging costs by informing risk managers about the least-cost risk management strategy. Wilson *et al.* (2007) illustrate how commodity processors such as bread baking and flour milling companies can use VaR to choose among competing risk management alternatives such as options, futures, forwards, hedging only production inputs, or hedging only outputs. It is noted that effective risk management strategies by agribusinesses also positively impact agricultural banks which finance farm production and margins as it reduces the likelihood of default. Furthermore, pursuant to the standards of the Basel Committee on Banking Supervision, banks – including agricultural banks – are required to use VaR to determine minimum capital reserves.

A significant share of the futures market is held by profit seeking speculators whose participation significantly enhances the liquidity and competitiveness of the market and offers the insurance sought by hedgers[6]. Sanders *et al.* (2008) document a sharp and sustained surge in activity by index and other large traders starting in 2006.

They find that the percent of total open interest attributable to index traders has ranged between 17 and 26 percent over 2006-2008 for CBOT wheat. Similarly, index traders' share of corn and soybeans has hovered around the 10-15 percent range during the same period. CBOT wheat, corn, and soybeans are the three main components of the Goldman Sachs Commodity Index (GSCI)'s agricultural sub-index.

Recent work has shown that consistently with the theory of normal backwardation, commodity futures yield positive excess returns, which in size are similar to excess returns earned on investment in US equities (Gorton and Rouwenhorst, 2006), and that commodity futures can serve as diversification instruments in conventional portfolios because of their low to negative correlations with equities and bonds (Gorton and Rouwenhorst, 2006; Jensen *et al.*, 2002). In this paper we use univariate and pooled time series of daily prices on corn, wheat, and soybean futures contracts from 2000 to 2008 to compute the value at risk for investment in these contracts. We do so using a nonparametric approach which is well suited for the typical characteristics of financial return series such as fat-tails, asymmetry, and serial dependence. Unlike equities and bond trades, a futures contract is a zero-sum game because for each trade there is a long position and a short position and a positive return for the holder of the long position represents a negative return for the holder of the short position. Given this characteristic of the commodity futures market, we gauge the market risk from the perspective of both buyers and sellers.

3.1 Data and descriptive statistics

The source of our data is the Commodity Research Bureau which compiles daily prices of commodity futures contracts since 1959. In this study we limit ourselves to corn, soybeans, and wheat contracts transacted between 2000 and 2008. For each of these commodities, there are several contracts of different maturity dates that are listed at

any point in time. To create a continuous time series of data, we compute the return series $\{r_t\}_{t=1}^T$ where $r_t = \log(P_t/P_{t-1}) \times 100$ using the daily prices for the contract that is closer to maturity unless the contract expires in the same month, in which case we roll into next shortest contract, as done by Gorton and Rouwenhorst (2006) and Brooks *et al.* (2005). This procedure yields 2,196 observations for corn futures, 2,183 observations for soybeans futures, and 2,200 observations for wheat futures. Figure 1 plots the returns from 2000 to 2008. Figure 2 plots the kernel density estimates of returns for corn, soybeans and wheat futures. The plot indicates that the returns exhibit volatility clustering which suggest that the variance of returns in heteroscedastic. As in Gouriéroux *et al.* (2000) and Chen and Tang (2005), we test for dependence in the squared returns by computing the χ^2 -distributed Ljung-Box test statistic $Q = T(T+2) \sum_{k=1}^{42} \gamma_k^2 / T - k$ where γ_k is the sample autocorrelation. The test statistics are $Q = 1612.40$ for corn, $Q = 1810.06$ for wheat, and $Q = 1446.09$ for soybeans. All three statistics are statistically significant at the 1 percent level; therefore confirming the presence of dynamics for all three returns series. Table I presents some descriptive statistics of the continuously compounded rate of return for each of the three commodities. The values for the skewness and kurtosis indicate that none of the three underlying distributions can be satisfactorily approximated by a normal distribution. We implement both the Jarque-Bera and Ait-Sahalia's (1996) tests to further examine the appropriateness of the normal approximation[7]. The tests statistics are listed in Table I; they indicate in all cases that the null of normality is rejected.

While sensible, this approach however is inefficient since it relies on only one futures contract at a time to compute the returns. There are several contracts of different maturities at any point in time for a given commodity. Therefore we use a second approach to construct the data, which consist of pooling the futures returns by choosing from the four contracts with the largest open interest. On average in our sample, the four contracts with the largest open interest account for more than 92 percent of all futures contracts. The approach proceeds as follows. First, for each of the three commodities, we construct the futures returns for the four contracts with the largest open interest. Second, we use Li's (1996) nonparametric test of similarity of unknown densities to test if the density of returns for the contract with the highest open interest is identical to each of three other densities[8]. If we cannot reject the null hypothesis that the densities are the same, then we pool the returns of these contracts and implement the nonparametric estimator based on the pooled data with the goal of increasing the efficiency of the nonparametric VaR estimate. Using the Li (1996) test, we fail to reject the null that for corn and wheat, the futures returns of the three contracts with the largest open interest have the same density; consequently, we pool the returns of three contracts. The pooled sample contains 6,411 observations for corn and 6,415 observations for wheat. For soybeans, the Li test fails to reject the null that all four densities are the same. The pooled sample for soybeans contains 8,465 observations. This pooling approach helps improve the precision of the Kernel VaR estimator by expanding the data to be locally averaged, especially in the tails of the distribution which are of particular interest.

3.2 Results and discussions

We implement the nonparametric Kernel method to quantify the daily VaR for the three futures contract return series for both long and short positions. Equation (2) is solved numerically using the quasi-Newton nonlinear optimization method (SAS subroutine *nlpqn*). We set $p=1$ percent and 5 percent to compute the VaR for both long and short sides of the market. To implement the Kernel VaR, a choice of $K(u)$ is required. Several

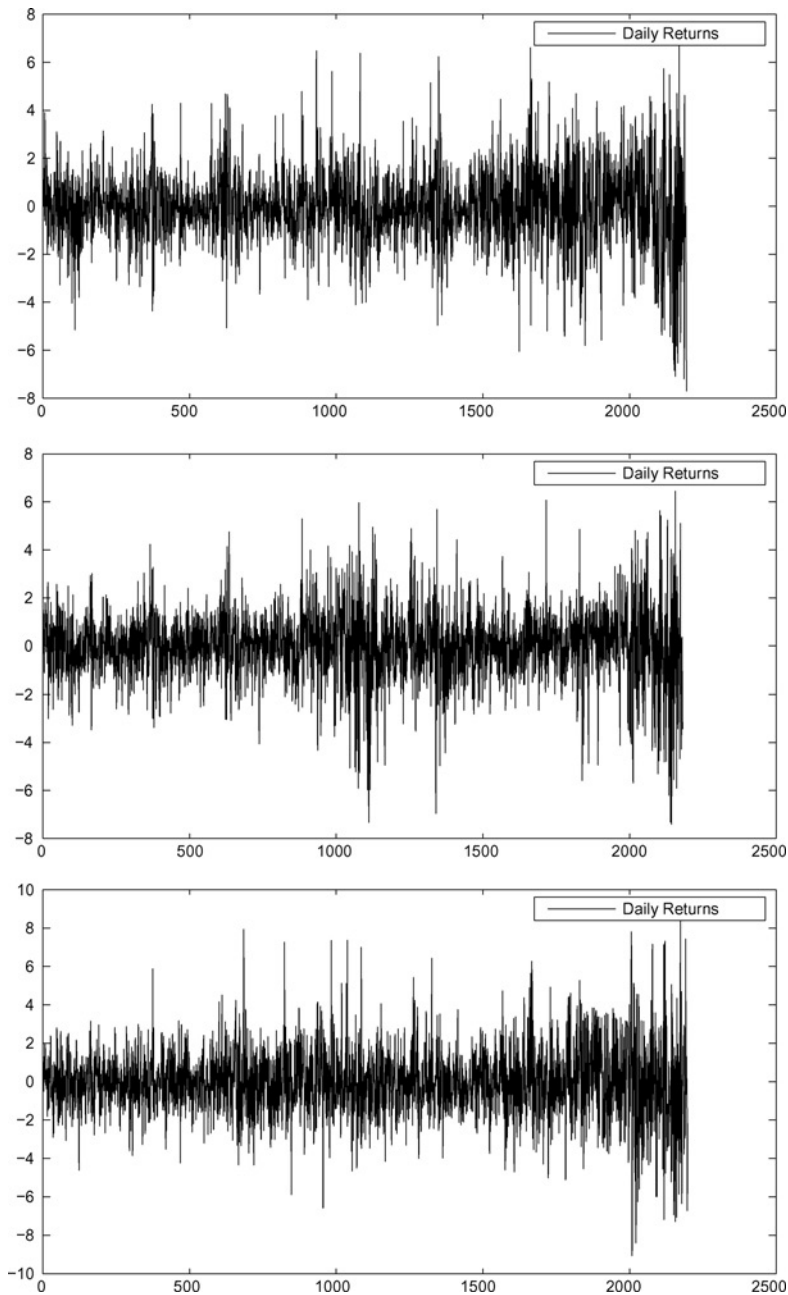


Figure 1.
Plots of the continuously
compounded returns for
corn, soybeans, and
wheat futures,
respectively, for the
period of 2000-2008

choices have been proposed in the literature[9]. We follow Gouriou *et al.* (2000) and use a Gaussian kernel to estimate the Kernel VaR. The smoothing parameter is selected by least squares cross-validation[10]. In all cases the value of equation (2) evaluated at the optimal VaR is smaller than 10^{-9} in absolute value.

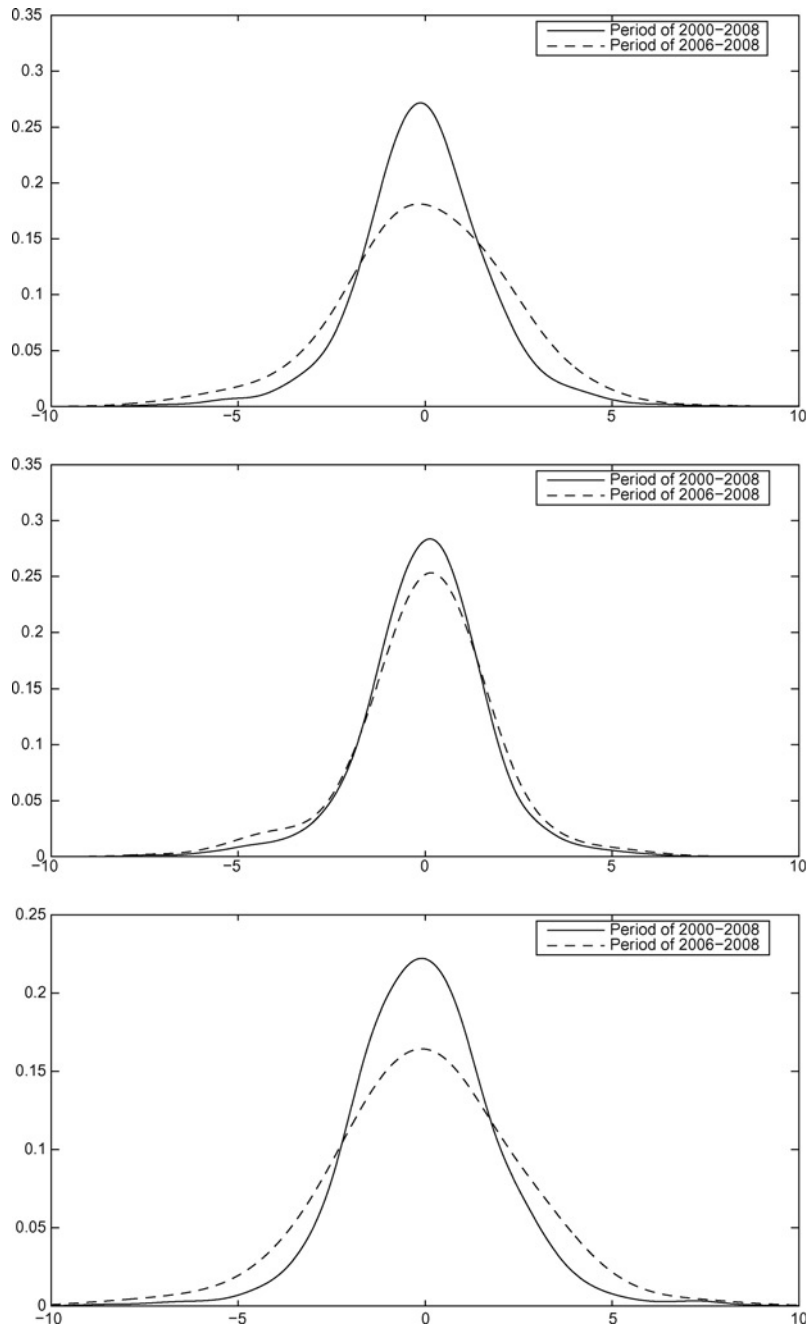


Figure 2.
Plots of the Kernel
density estimates of
returns for corn,
soybeans, and wheat
futures, respectively

Table II presents the results of the nonparametric Kernel and VaR estimations for the 95 and 99 percent confidence levels without the negative signs. The standard errors of the VaR estimates are also reported[11]. We present two sets of nonparametric

estimates. The first set (univariate series) is obtained by using only one time series of returns based on the prices of the shortest contract as explained in the data section. The second set (pooled series) is obtained by pooling the returns of futures contracts with different maturities.

A number of findings should be stressed. First, in all cases, the VaR based on the univariate return series is always larger in absolute value than the VaR based on the pooled time series. Furthermore, the pooled VaR falls outside of the 95 percent confidence interval of the univariate VaR for all cases considered in Table II. This suggests that the reliance on just the returns of the shortest futures contract over-estimates the degree of market risk by a statistically and economically significant value. Second, wheat futures are the riskiest of the three we consider in this paper with VaR estimates that are significantly larger than estimates for corn and soybeans futures; soybeans futures have the smallest market risk. Third, for corn and soybeans, long positions are riskier than short positions for both the 95 and 99 percent confidence levels while the opposite is observed for wheat futures. This result corroborates the evidence of negative skewness for corn and soybeans futures and positive skewness for wheat futures presented in Table I.

To have a better understanding of the dynamics in the data, we estimate the nonparametric density function using daily returns only for the period 2006 to 2008. This period is characterized by rising commodity futures prices as well as increased price volatility (Food and Agriculture Organization, 2006). The standard deviations

Contract	Corn	Soybeans	Wheat
Mean (%)	-0.0587	0.0054	-0.0351
Standard deviation	1.686	1.602	1.926
Skewness	-0.0778	-0.3324	0.1019
Kurtosis	4.8024	5.1174	4.9617
Jarque-Bera statistic	299.45	448.01	356.56
Critical value	5.96	5.96	5.96
Ait-Sahalia statistic	181.16	169.11	225.97
Critical value	4.51	4.42	4.81

Table I.
Summary statistics for
the data

Confidence level (1 - p)	Corn		Soybeans		Wheat	
	95%	99%	95%	99%	95%	99%
<i>(A) Long positions</i>						
Univariate series	2.865	4.934	2.735	4.889	3.083	5.109
(standard error)	(0.017)	(0.049)	(0.017)	(0.044)	(0.016)	(0.055)
Pooled series	2.629	4.584	2.582	4.641	2.785	4.985
(standard error)	(0.016)	(0.047)	(0.016)	(0.041)	(0.016)	(0.058)
<i>(B) Short positions</i>						
Univariate series	2.766	4.502	2.556	4.280	3.207	5.265
(standard error)	(0.016)	(0.039)	(0.015)	(0.042)	(0.017)	(0.056)
Pooled series	2.574	4.204	2.407	4.131	2.876	4.835
(standard error)	(0.015)	(0.037)	(0.014)	(0.041)	(0.016)	(0.052)

Table II.
Nonparametric value at
risk estimates for corn,
soybeans, and wheat
futures contracts for the
period 2000-2008

over this period (2.171, 1.806, and 2.455 percent, respectively for corn, soybeans, and wheat futures) are larger than standard deviations over the 2000 to 2008 period. Figure 1 shows that the density estimate based on 2006 to 2008 daily data has “fatter” tails than that based on the entire study period for all three commodities. This captures an increase in riskiness of futures returns. Table III displays the nonparametric VaR estimates based on the 2006-2008 distribution. It clearly indicates a significant increase of market risk with VaRs that are 12 to 41 percent (10 to 43 percent) larger than the corresponding VaRs in Table II for the univariate (pooled) method. Wheat futures have experienced the highest increase in market risk over the three-year period with VaRs that are between 24 and 40 percent (25 and 43 percent) larger. Conversely, market risk for soybeans futures has increased at a more moderate pace with the VaR increasing by between 12 and 26 percent (10 and 27 percent).

4. Summary and conclusions

VaR has become a standard measure of market risk thanks in large part its simplicity as a concept, the development by Riskmetrics of an open source VaR methodology, and the decision of the Basel Committee and the Federal Reserve to allow banks to use their internal VaR to determine capital requirements. While the finance literature is abundant with theory and application of VaR methods, few papers have applied the concept of VaR in the context of agricultural finance. This paper implements a nonparametric Kernel approach to the computation of the VaR for investments in three agricultural commodity futures: corn, soybeans, and wheat. Unlike the standard (parametric) VaR methods, the Kernel method is exempt of any functional form misspecification; it accommodates serial dependence, asymmetry, and fat tails. The results of the empirical application indicate that wheat futures are riskier than either corn or soybeans futures over both periods considered in this paper – 2000-2008 and 2006-2008 – and that all three commodities have experienced a significant jump in market risk over the 2006-2008 period with an increase of 12 to 41 percent or 10 to 43 percent – depending on the method – of the VaR estimates relative to the long-run estimates.

This paper also illustrates practical potential of the Kernel approach for the measurement of market risk in several other agricultural contexts besides futures contracts such as risk management for cattle feeders (Manfredo and Leuthold, 2001), hog production (Odening and Hinrichs, 2003), harvest-time revenue (AgRisk, The Ohio State University), and commodity processing (Wilson *et al.*, 2007).

Confidence level (1 - p)	Corn		Soybeans		Wheat	
	95%	99%	95%	99%	95%	99%
<i>(A) Long positions</i>						
Univariate series	4.042	6.335	3.460	5.504	4.316	7.082
(standard error)	(0.022)	(0.049)	(0.022)	(0.044)	(0.023)	(0.063)
Pooled series	3.633	5.768	3.280	5.369	3.996	6.595
(standard error)	(0.021)	(0.045)	(0.022)	(0.046)	(0.023)	(0.051)
<i>(B) Short positions</i>						
Univariate series	3.704	5.343	2.899	4.903	4.280	6.540
(standard error)	(0.018)	(0.042)	(0.016)	(0.045)	(0.021)	(0.059)
Pooled series	3.364	4.895	2.657	4.787	3.774	6.042
(standard error)	(0.017)	(0.039)	(0.016)	(0.048)	(0.018)	(0.064)

Table III.
Nonparametric value at risk estimates for corn, soybeans, and wheat futures contracts for the period 2006-2008

Notes

1. Another VaR estimation methods include the historical simulation and the Monte Carlo simulation methods. Both methods seek to estimate the portfolio distribution instead of assuming its functional form. The Monte Carlo simulation approach (Linsmeier and Pearson, 1996; Jorion, 2000) consists of drawing random values of the risk factors of the portfolio from a specified joint parametric distribution. This process is repeated thousands of times. The generated random values are used to construct portfolio returns with which are then ordered from smallest to largest. The VaR estimate is simply the quantile of the ordered series, assuming returns have zero mean, that corresponds to the specified confidence level. This method is just as prone to specification error as the variance-covariance method since a joint distribution is required. Choosing a data generating process is considerably more difficult for complex portfolios that contain a large number of correlated risk factors. Historical simulation is implemented by finding the order statistic of the empirical distribution of past portfolio values that corresponds to the desired confidence level.
2. The reader is directed to Chen and Tang (2005) for extensive discussion of the underlying assumptions and derivation of the asymptotic properties of the Kernel estimator.
3. Grain elevator operators play an important intermediary role in agricultural commodity markets by buying grain from farmers and storing and selling it to commodity processors, exporting firms, and livestock feeders among others. As per the 2002 Economic Census, grain elevators operated in almost 6,000 locations, had over 61,000 employees, and generated almost \$90 billion in sales. The reader is directed to "Can Grain Elevators Survive Record Crop prices" by the Federal Reserve Bank of Kansas City (2008) for additional facts about the grain elevator business.
4. The surge in agricultural commodity prices in 2008 led to large margin calls for grain elevators, jeopardizing their cash positions and causing some to increase their lines of credit substantially. Some operators simply filed for bankruptcy as they were unable to extend their credit lines in order to comply with margin requirements (Federal Reserve Bank of Kansas City, 2008).
5. To implement the nonparametric Kernel method to compute a weekly VaR, weekly data must be used as the time scaling formula generally used to convert the daily VaR to a longer-horizon VaR is only valid when the returns are *i.i.d* normal which is generally not a tenable assumption for financial data.
6. The launching of the tradable indices by Goldman Sacks (Goldman Sachs Composite Commodity Index) in 1991 and the Dow Jones-AIG commodity index in 1998, among others, has fueled increased futures trading by speculators.
7. The Ait-Sahalia test consists of evaluating the integrated squared distance between the normal density and the nonparametric Kernel density estimator for each of the three return series. Ait-Salalia (1996) shows $\sqrt{h}[\sum_{t=1}^T (\phi(r_t) - \hat{f}(r_t))^2]$ follows the normal distribution with mean $(\int_{-\infty}^{+\infty} K^2(x)dx)(1/T \sum_{t=1}^T \hat{f}(r_t))$ and variance $2(\int_{-\infty}^{+\infty} [\int_{-\infty}^{+\infty} K(u)K(u+x)du]^2 dx) (1/T \sum_{t=1}^T \hat{f}^3(r_t))$ where $\phi(\cdot)$ is the normal density function.
8. Suppose we have T_1 and T_2 observations, respectively, on two random variables x and y with unknown densities $f(x)$ and $g(y)$. Let $\lambda = (T_1/T_2)$, $K_{ij}^x = K(x(i) - x(j)/h)$, and $K_{ij}^{x,y} = K(x(i) - y(j)/h)$. The Li (1996) test statistic for the null hypothesis $H_0: f(x) = g(x)$ is: $T\sqrt{h}I_2/\hat{\sigma} \rightarrow N(0,1)$, where $I_2 = h^{-1}(\sum_i \sum_{j \neq i} [K_{ij}^x/(T_1(T_1 - 1)) + K_{ij}^y/T_2((T_2 - 1)) - K_{ij}^{x,y}/(T_1(T_2 - 1)) - K_{ij}^{y,x}/(T_1(T_2 - 1))])$ and $\hat{\sigma}^2 = 2(\sum_i \sum_j [(K_{ij}^x/T_1^2) + (K_{ij}^y\lambda^2/T_2^2) + (K_{ij}^{x,y}\lambda/(T_1T_2)) + (K_{ij}^{y,x}\lambda/(T_1T_2))]) \int K^2(u)du$

9. The Kernel function is generally chosen to be a symmetric unimodal density. Wand and Jones (1995, p. 31) show that the Epanechnikov function is the optimal Kernel function in that it leads to lowest asymptotic mean integrated squared error (AMISE) for the nonparametric density estimator $\hat{f}(r)$. However, they also show that using other kernel functions that satisfy the requirements listed in Assumption 1 of Chen and Tang (2005) such as the Biweight, the Triweight, or the Gaussian kernel function leads to only a negligible loss in efficiency. This result follows from the Law of Large Numbers as the density estimator consists of averaging smoothed points based on a large sample size. We follow Gouriéroux *et al.* (2000) and use a Gaussian kernel to estimate the Kernel VaR.
10. The least squares cross-validation procedure consists of minimizing the loss function $\int (\hat{f}(r) - f(r))^2 dr$ with respect to h , which amounts to minimizing $T^{-2}h^{-1} \sum_{t=1}^T \sum_{t'=1}^T KoK(r_t - r_{t'}/h) - 2/T \sum_{t=1}^T \hat{f}_{-t}(r_t)$ with respect to h where $\hat{f}_{-t}(r_t)$ is the "leave-one-out" Kernel density estimator obtained by omitting the t th observation (Pagan and Ullah, 1999). However, given the evidence of dependence in the data, we follow standard practice (e.g. Sam and Jiang, 2009) and remove a block of one-month of data instead of just one daily observation. We also experimented with removing two weeks and two months of data, respectively, with no material impact on the value of the smoothing parameter.
11. The standard errors are computed using the SAS optimization subroutine "nlpdfd", which is a module that numerically approximates the hessian.

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